# Climate Policy and Innovation: A Quantitative Macroeconomic Analysis
# Online Appendix

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## A. Derivations of the main equations

I derive the main equations in the text. For ease of presentation, some of the equations are repeated. The final goods producer chooses $F$, $G$, $N$, and $O^\star$ to maximize profits taking prices as given. His optimization problem is (equation (9) in the text)

$$\text{(A1)} \qquad \max_{F_t, G_t, N_t, O_t^\star} \left\{ Y_t - (P_{ft} + \tau_f)F_t - P_{gt}G_t - (P_{ot}^\star + \tau_o)O_t^\star - P_{nt}N_t \right\},$$

subject to the production technology defined in equations (1) and (2) in the text. The first order conditions imply the relative demands for the intermediate inputs are inversely related to their prices,

$$\text{(A2)} \qquad \frac{G_t^d}{\tilde{F}_t^d} = \left(\frac{P_{\tilde{F}t}}{P_{gt}}\right)^{\varepsilon_e}, \qquad \frac{F_t^d}{(O_t^\star)^d} = \left(\frac{P_{ot}^\star + \tau_o}{P_{ft} + \tau_f}\right)^{\varepsilon_f} \left(\frac{\delta_{\widetilde{F}}}{1 - \delta_{\widetilde{F}}}\right)^{\varepsilon_f},$$
$$\frac{E_t^d}{N_t^d} = \left(\frac{P_{nt}}{P_{et}}\right)^{\varepsilon_y} \left(\frac{\delta_y}{1 - \delta_y}\right)^{\varepsilon_y}.$$

Variables $P_{\widetilde{F}}$ and $P_e$ denote the tax-inclusive prices of optimally chosen composites $\widetilde{F}$ and $E$, respectively. The first order and zero profit conditions imply that these prices are

$$\text{(A3)} \qquad P_{\widetilde{F}t} = (\delta_{\widetilde{F}}^{\varepsilon_f}(P_{ft} + \tau_f)^{1-\varepsilon_f} + (1 - \delta_{\widetilde{F}})^{\varepsilon_f}(P_{ot}^\star + \tau_o)^{1-\varepsilon_f})^{\frac{1}{1-\varepsilon_f}}$$
$$P_{et} = (P_{\tilde{F}t}^{1-\varepsilon_e} + P_{gt}^{1-\varepsilon_e})^{\frac{1}{1-\varepsilon_e}}.$$

The final good is the numeraire. I normalize its price to unity. This yields the ideal price index

$$\text{(A4)} \qquad \delta_y^{\varepsilon_y} P_{et}^{1-\varepsilon_y} + (1 - \delta_y)^{\varepsilon_y} P_{nt}^{1-\varepsilon_y} = P_{yt} \equiv 1.$$

The intermediate-goods producers make fossil, green, and non-energy inputs which they sell to the final-good producer. I discuss the equations with respect to a representative fossil-energy producer; the other sectors are symmetric. The fossil-energy producer chooses labor and machines to maximize profits taking prices as given,

$$\max_{L_{ft},X_{fit}} P_{ft}L_{ft}^{1-\alpha_f}\int_0^1 X_{fit}^{\alpha_f}A_{fit}^{1-\alpha_f}\,di - w_{lft}L_{ft} - \int_0^1 P^x_{fit}X_{fit}\,di. \tag{A5}$$

Variable $P^x_{fi}$ denotes the price of machine $i$ in sector $f$. The first order condition for $X_{fit}$ implies the demand for machines

$$X_{fit} = \left(\frac{\alpha_f P_{ft}}{P^x_{fit}}\right)^{\frac{1}{1-\alpha_f}} A_{fit}L_{ft}, \tag{A6}$$

where value $1/(1-\alpha_f)$ is the price elasticity of demand for machines. The first order condition for $L_{ft}$ implies the wages to workers in sector $f$,

$$w_{lft} = (1-\alpha_f)P_{ft}X_{ft}^{\alpha_f}L_{ft}^{-\alpha_f}A_{ft}^{1-\alpha_f}. \tag{A7}$$

In equilibrium, labor market clearing requires that workers' wages are equated across all sectors, $w_{lft} = w_{lgt} = w_{lnt}$, and that total labor demand equal the fixed, exogenous supply, $L_{ft} + L_{gt} + L_{nt} = L$.

The machine producers make machines which they sell to the intermediate-goods producers. The machines embody technology. Each machine, regardless of the sector and the level of technology, costs one unit of final good to produce. Each machine producer chooses price, quantity of machines, and the number of scientists to maximize profits. The optimization problem for fossil-energy machine producer $i$ in period $t$ is

$$\max_{P^x_{fit},X_{fit},S_{fit}} P^x_{fit}X_{fit} - X_{fit} - w_{sft}S_{fit} \tag{A8}$$

subject to the demand for machines, (equation (A6)) and the evolution of technology (equation (4) in the text).

The first order condition for the number of machines implies that the optimal machine price is a constant markup over marginal cost

$$P^x_{fit} = \frac{1}{\alpha_f}. \tag{A9}$$

This constant markup arises because the price-elasticity of machine demand, $1/(1-\alpha_f)$, is constant. Increases in the machine share increase the demand elasticity and decrease the markup.

Finally, the first order conditions for the number of scientists imply that the wage to a scientist in sector $f$ is

$$w_{sft} = \frac{\eta\gamma A_{ft-1}\left(\frac{A_{t-1}}{A_{ft-1}}\right)^{\phi} P^{x}_{fit}X_{fit}}{\rho_f^{\eta}\left(\frac{1}{1-\alpha_f}\right)S_{ft}^{1-\eta}A_{fit}}. \tag{A10}$$

Since the market for scientists is perfectly competitive, the wage of a scientist in the fossil energy sector equals the marginal return to innovation in that sector. The equilibrium is symmetric across all machine producers within a sector, and, hence, $P^x_{fit}X_{fit} = P^x_{ft}X_{ft}$. To derive equation (10) in the text, observe that equation (A6) combined with the production for fossil energy (equation (3) in the text) imply that $P^x_{ft}X_{ft} = \alpha_f P_{ft}F_t$. Substitute this relationship into equation (A10) to get equation (10) in the text.

## B. Time period and within-sector spillovers

The time period in the model is five years. This choice implies that technology spillovers within a sector occur in five years. To determine this time period, I examine the rate of technology spillovers experienced in solar power (a green industry) and in offshore drilling (a fossil industry). In both cases, within-sector technology spillovers frequently occur in less than five years.

One form of technology embodied in a solar cell is the cell's efficiency. Cell efficiency measures the ratio of the cell's electrical output to incident energy from sunlight. Higher cell efficiency corresponds to higher technology. Figure B1, from the National Renewable Energy Laboratory, plots advances in cell efficiency from 1970-2010 and the company or research institution that achieved the advance. In most cases, the company with the leading cell efficiency is surpassed by a different company within five years. For example, in 1970, Mobile Solar had the leading efficiency in Single crystal non-concentrator Si cells. In 1978, Renewable Capital Assets (RCA) passed Mobile Solar; in 1980, Sandia National Laboratory passed RCA, and so on. The average length of time that a company or research institution maintains the leading efficiency is 3.84 years. This leapfrogging occurs in less than five years, on average, suggesting that within-sector spillovers over a five-year period are reasonable in the case of solar electricity.

As an example from the fossil energy sector, I consider the development of offshore drilling technology. An early technological advance in the offshore industry occurred in 1954, when the Offshore Drilling and Exploration Company (ODECO) developed the first submersible drilling barge, "Mr. Charlie." Mr. Charlie was designed to drill in what was considered deep water at the time (30 feet). By 1957, just three years after Mr. Charlie's introduction, there were 23 such units in operation in the Gulf and 14 more under construction by numerous oil companies, including Zapata Offshore Company (founded by George H.W.

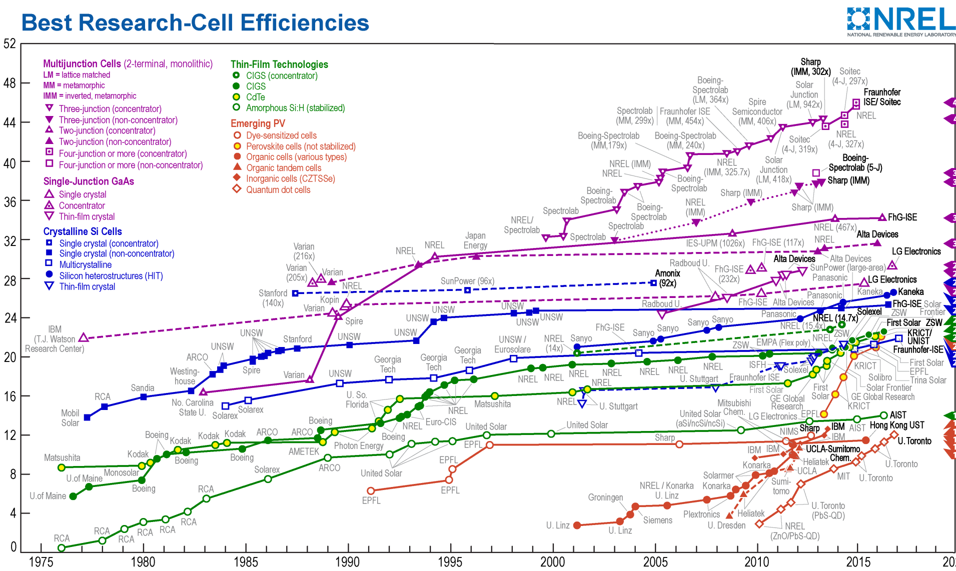


Figure B1. Leading Solar Cell Efficiencies

*Source:* Courtesy of the National Renewable Energy Laboratory (NREL), Golden, Colorado

Bush), ODECO, and others. Thus, in less than five years, the technology embodied in ODECO's Mr. Charlie spilled over to other major players in the industry (Schempf (2007); Oil Spill Commission (2011)).

A second major development in offshore drilling occurred in 1962, when Shell Oil launched Blue Water 1, a semi-submersible floating drilling platform that was equipped to operate in up to 600 feet of water (previous platforms could not exceed 150 feet). However, when Shell tried to lease the land for drilling, it was the only bidder on some of the deepwater tracts, and the government refused to honor bids without competition. Since no other companies could operate at those depths, "[Shell] realized that the only way [it] could ever have access to those frontier areas was to share [its] knowledge with the rest of the industry, to give them a base of technology from which they could expand" (Ron Geer, Shell mechanical engineer). In 1963, Shell hosted a "school for industries" in which it shared its frontier deep water technology with seven other companies. By 1968, these within-sector spillovers had led to the construction of 23 Blue-Water-like semi-submersibles and opened up deeper and rougher areas of the ocean to oil drilling and exploration (Priest (2007)).

Shell Oil continued its advance into deeper waters and, in 1976, constructed "Cognac," a fixed platform connected to a well in 1000 feet of water in the Gulf. At the time, Cognac was the most costly and technologically advanced fixed platform installation ever completed. But within five years of its construction, other companies innovated on Cognac's design and built similar platforms for much less money. To emphasize its cost savings compared to Cognac, Union Oil named its two 1000-foot platforms constructed from 1980-1981, "Cerveza" and "Cerveza-light." But as energy historian Tyler Priest notes, "these beer-budget projects could not have happened without the deep water precedent established by Cognac" (Priest (2007)). Again, the development and diffusion of offshore drilling technology suggests that within-sector spillovers often occur in less than five years.

### C. Robustness with respect to the method of moments

I consider two robustness checks with respect to the method-of-moments calibration procedure. First, as is standard in much of the macroeconomic literature, I evaluate the fit of the model on five moments that were not directly targeted: the percent change in the average real GDP per capita between the last 5 year period on the balanced growth path (1966-1970) and the shock period (1971-1975), the fraction of total labor in the fossil energy sector and the fraction of total capital (fixed assets) in the fossil energy sector in both the balanced growth path and the shock period. While matching these moments is not as crucial for capturing the innovation incentives as matching the targeted moments, the model's performance with respect to these non-targeted moments provides one measure of the model's fit to the data. Table C1 reports the results from this robustness exercise. The values of these moments are relatively similar in the model and the data, suggesting that the model's fit is reasonably strong.

Table C1—Non-Targeted Moments

| | Time Period | Model Value | Empirical Value |
|---|---|---|---|
| Fraction of labor in fossil | 1961-1970 | 0.01 | 0.01 |
| | 1971-1975 | 0.01 | 0.01 |
| Fraction of capital in fossil | 1961-1970 | 0.05 | 0.03 |
| | 1971-1975 | 0.05 | 0.03 |
| %$\Delta$ RGDP in per capita | - | 9.16 | 9.95 |

*Source:* The empirical values are based on the author's calculations using data from the BEA. The model values are based on the author's calculations from simulations of the calibrated model.

I consider a second robustness check of the method-of-moments procedure to address concerns that one limitation of the calibration strategy is that the early 1970s oil shocks occurred forty years ago. It is possible that some of the parameter values could have changed over time. To mitigate this concern, I compare the responsiveness of innovation to the 2003 oil shock in the data and in the calibrated model. I do not change the calibration of the model in this exercise; the parameter values are the same as those listed in Table 1. The goal of this exercise is to determine if the model calibrated to the 1970s oil shock matches the innovation response to the 2003 oil shock.

As with the early 1970s oil shocks, I begin the simulation on a balanced growth path and then introduce an oil shock. I choose the size of the shock to match the average increase in the price of oil imports during the 2003-2007 time period from the previous five year period (1998-2002). I calculate the percent change in fossil and green innovation relative to the percent change in the oil import price in both the simulation and in the data. I refer to this quantity as the elasticity of innovation with respect to the price of oil imports.[1] Table C2 reports these elasticities in the model and in the data.

Table C2—Elasticity of innovation with respect to the oil import price

| | Model | Data |
|---|---|---|
| Fossil Innovation | 0.8 | 0.7 |
| Green Innovation | 0.5 | 0.7 |
| Total Energy Innovation | 0.7 | 0.7 |

*Source:* The empirical values are based on the author's calculations using data from the EIA and the NSF Survey of Industrial Research and Development. The model values are based on the author's calculations from simulations of the calibrated model.

The empirical and model estimates are similar, suggesting that the parameters governing the responsiveness of energy innovation to price changes have not changed substantially over time. The largest discrepancy between the model and the data is that the price elasticity of green innovation is lower in the model. This difference could be partly explained by policies or expectations of policies which encouraged green innovation during this time. For example, over this period, an increasing number of states adopted renewable portfolio standards, and congress and the president began to lay the groundwork for the Energy Independence and Security Act of 2007. Both of these policy developments would encourage green innovation and lead to a higher empirical price elasticity.

This exercise has two caveats. First, energy prices and energy innovation are not stable for a sustained period preceding the 2003 oil shock, suggesting that the assumption that the economy was on a long-run balanced growth path prior to the shock is imperfect. Second, following the 1970s, agents learned that energy prices are uncertain and they formed expectations over future energy prices. I do

[1] A rigorous empirical estimate of the elasticity fossil and green energy innovation with respect to a change in the oil import price is beyond the scope of this paper. The goal of this robustness check is to show that the model generally matches a back-of-the-envelope calculation of the innovation responses to oil price changes in the more recent time periods.

not model expectations in this robustness analysis. The advantage of calibrating the model using the early 1970s' oil shock instead of the 2003 oil shock is that it avoids both of these caveats. Energy prices were relatively flat for the 20 years prior to the early 1970s oil shock, suggesting the economy could plausibly have been on a balanced growth path and that agents did not anticipate the shock.

## D. Standard error estimates

I jointly calibrate the parameters $\{\alpha_g, \varepsilon_f, \delta_{\widetilde{F}}, \delta_y, \eta, \gamma\}$ and the productivity shock, $\nu$, to capture the relationships between energy prices, production, and innovation. To obtain empirical evidence of these relationships, I analyze the energy price increases triggered by historical oil shocks in the early 1970s. Specifically, I calibrate the parameters to match moments generated by the oil and productivity shocks of the early 1970s in the US economy with moments generated by the following experiment in the model:

**Initial balanced growth path (1961-1970):** The economy is on a balanced growth path with respect to the price of oil imports and environmental and energy policies.

**Shock period (1971-1975):** Two unexpected shocks realize: (1) the price of oil imports increases from its value on the balanced growth path; and (2) a negative productivity shock affects domestic fossil energy production.

I target two moments from the initial balanced growth path: (1) average fossil energy production as a share of GDP and (2) the average value of oil imports as a share of GDP. I target four moments from the shock period: (1) average fossil energy production as a share of GDP, (2) the average value of oil imports as a share of GDP, (3) average fossil energy R&D expenditures as a fraction of total R&D expenditures, and (4) average green energy R&D expenditures as a fraction of total R&D expenditures. Table 2 reports the empirical values of these moments. Additionally, I target the long-run annual growth rate of GDP per capita of 2 percent. See Section IV for more details on these moments and their selection process. I choose the parameter values that minimize the sum of the square of the residuals between the empirical and model values of the moments.

The provision of valid standard errors for my parameter estimates requires a model of the data generating process for these moments. This is difficult as the moments are realized only once and my model provides no guidance on their stochastic generating process. Furthermore, while I can reasonably assume that the values of the moments on the balanced growth path are constant from 1961-1970, this assumption is clearly invalid during the shock period, 1971-1975. Mindful of these concerns, I take the view that the standard errors should reflect uncertainty in my parameter estimation induced by idiosyncratic variation in the measured values of my moments. I propose a resampling procedure that uses annual variation over the balanced growth path to approximate this.

Table D1—Parameter Estimates with Standard Errors in Parentheses

| | |
|---|---|
| Green energy machine share: $\alpha_g$ | 0.9<br>( 0.04) |
| Diminishing returns: $\eta$ | 0.8<br>( 0.24) |
| Scientist efficiency: $\gamma$ | 4.0<br>( 3.50) |
| Fossil elasticity of substitution: $\varepsilon_f$ | 6.2<br>( 3.11) |
| Distribution parameter: $\delta_y$ | 1.4e-38<br>( 5.0e-38) |
| Distribution parameter: $\delta_{\tilde{F}}$ | 0.5<br>( 0.05) |
| Productivity shock: $\nu$ | 0.6<br>( 0.05) |

Specifically, I interpret any annual variation in the value of the balanced growth path moments as random noise. I resample each balanced growth path moment as its average value during the balanced growth path plus a random error drawn from a normal distribution with mean zero and standard deviation equal to the standard deviation of the annual value of the moment along the balanced growth path.

I interpret any annual variation in the value of the shock period moments as both random error and the economic response to the shocks of the early 1970s. However, if each shock period moment was instead calculated over the balanced growth path period, its annual value should be constant. Thus, I resample each shock period moment as its average value during the shock period plus a random error drawn from a normal distribution with mean zero and standard deviation equal to the standard deviation of the annual value of the moment along the balanced growth path. Data on fossil and green R&D are not available during the balanced growth path period. For these two moments, I use the standard deviation of the fraction of total R&D in the petroleum refining and extraction sectors (SIC codes 13 and 29) on the balanced growth path instead of the standard deviation of the fractions of fossil and green R&D.[2,3]

I resample the moments and reestimate the parameters 300 times. I then compute the bootstrapped standard error of each estimated parameter.[4] Table D1 reports the parameter values for the baseline specification with the bootstrapped standard errors in parentheses. With the exception of $\gamma$ and $\delta_y$, the parameter estimates demonstrate a reasonable degree of precision. However, these standard errors should be interpreted with caution. Their computation relies critically on the assumption that the idiosyncratic variation in the moments during the balanced growth path period is the same as during the shock period. Moreover, annual data over a ten year period is not necessarily representative of the true variance in the empirical distribution of these moments.

## E. Target stringency and time frame

A key finding is that the carbon tax necessary to achieve the emissions target is 19.2 percent lower when innovation is endogenous. However, this result is sensitive to both the size of the targeted reduction in emissions and the time frame in which the reduction must be achieved. In this section, I analyze the effects of innovation for different-sized emissions targets and different time frames.

The left panel of Figure E1 plots the percent reduction in the carbon tax from endogenous innovation for different-sized emissions targets. For example, the carbon tax required to achieve a 10-percent reduction in emissions is 21 percent lower when innovation is endogenous. The effect of endogenous innovation on the size of the carbon tax falls as the stringency of the emissions target increases (e.g., as the target goes from a 10-percent to a 20-percent reduction in emissions). Even with large changes in innovation, the relative technology stocks evolve slowly. A more stringent emissions target forces agents to rely less on technological advances and more on shifts in production factors (i.e., workers and machines) to achieve the target. This switch reduces the role of endogenous innovation and its accompanying effects on the carbon tax.

The right panel of Figure E1 plots the percent decrease from endogenous innovation under a carbon tax designed to achieve a 30-percent reduction in emissions over different time periods. For example, the carbon tax required to achieve a 30-percent reduction in emissions in 25 years is 22 percent lower when innovation is endogenous. The reduction in the carbon tax from endogenous innovation increases with longer time frames. Again, even with large shifts in innovation, changes in the relative technology stocks occur gradually. A longer time frame provides more time for technological change to occur and thereby allows agents

[2] I rescale the estimate of the standard deviation of the fraction of petroleum R&D to account for small differences in the means between the fractions of fossil and green energy R&D during the shock period and the fraction of petroleum R&D on the balanced growth path.

[3] I do not resample the long-run growth rate of per capita GDP.

[4] For 76 of the 300 sets of moments, there did not exist parameter values such that the model was consistent with the set of moments. I removed these draws from the calculation of the standard error.

to rely more heavily on innovation to reduce emissions.

Climate policy simulation models that exogenously assume large advances in green technological progress typically obtain lower carbon tax estimates for a given abatement target (Pew Research Center (2010)). As long as the time frame for a given emissions target is sufficiently long, the results of this paper suggest that such technological advances are plausible and could lead to considerable reductions in the carbon tax. However, if policy makers strive to achieve large reductions in emissions quickly, then the potential for innovation to reduce the carbon tax is relatively small.

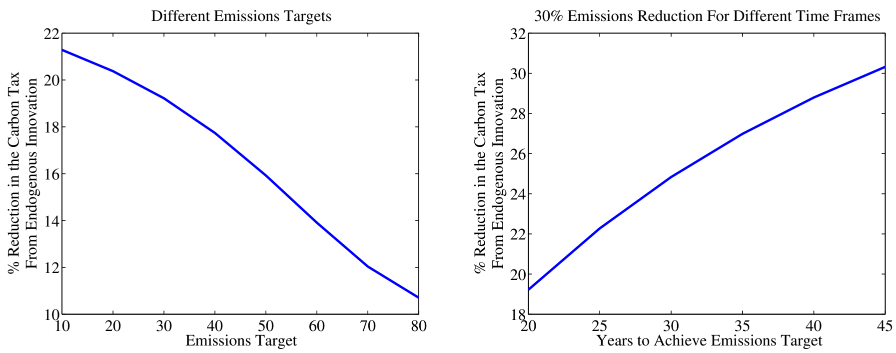


Figure E1. Effects of Innovation on the Size of the Carbon Tax

## F. Parameter sensitivity

### *F1. All model parameters*

Table F1 reports sensitivity analysis with respect to all the model parameters. As a summary statistic for the analysis, I analyze the percent that endogenous innovation reduces the size of the carbon tax required to achieve the emissions target.[5] The central column in Table F1 reports this summary statistic when the parameters equal their calibrated in values in the main specification (given in Table 1 in the text). The high (low) column reports this summary statistic when the parameter is 25 percent bigger (smaller) then its value in Table 1. For example, reading from the first line of Table F1, if $\varepsilon_y$ is 25 percent larger than its value in Table 1, then endogenous innovation reduces the carbon tax required to achieve the emissions target by 19.0 percent.[6]

[5]Tables F2 and F3 in Appendix F.F3 report the levels of the tax for each parameter perturbation. For most of the parameter values, the levels are not substantially different from the main specification.

[6]Note that I only consider the high case for $\varepsilon_y$. The main specification uses $\varepsilon_y = 0.05$. Values of $\varepsilon_y < 0.05$ introduce kinks into the equilibrium conditions, making it very difficult to solve the model.

Table F1—Percent Change in the Carbon Tax from Endog. Innovation

| Parameter | High | Central | Low |
|---|---|---|---|
| *Imposed parameters* | | | |
| Output elasticity of substitution: $\varepsilon_y$ | 19.0 | 19.2 | - |
| Cross-sector spillovers: $\phi$ | 18.4 | 19.2 | 20.2 |
| *Direct from data series* | | | |
| Machine share in fossil energy: $\alpha_f$ | 13.4 | 19.2 | 8.7 |
| Machine share in non-energy: $\alpha_n$ | 26.2 | 19.2 | 4.9 |
| Energy elasticity of substitution:$\varepsilon_e$ | 20.3 | 19.2 | 17.4 |
| Sector diversity: $\rho_f$ | 19.5 | 19.2 | 18.9 |
| Sector diversity: $\rho_g$ | 19.1 | 19.2 | 19.3 |
| Population of scientists: $S$ | 19.2 | 19.2 | 19.2 |
| *Method of moments* | | | |
| Machine share in green energy: $\alpha_g$ | 20.2 | 19.2 | 26.6 |
| Diminishing returns: $\eta$ | 35.9 | 19.2 | 10.9 |
| Scientist efficiency: $\gamma$ | 21.7 | 19.2 | 16.0 |
| Fossil elasticity of substitution: $\varepsilon_f$ | 18.3 | 19.2 | 20.1 |
| Distribution parameter: $\delta_y$ | 19.2 | 19.2 | 19.2 |
| Distribution parameter: $\delta_{\widetilde{F}}$ | 17.0 | 19.2 | 32.9 |

*Note:* The central column reports the result when the parameter equals its calibrated value in the main specification (given in Table 1). The high (low) column reports the result when the parameter is 25 percent bigger (smaller) then its value in Table 1. The one exception to this rule is for $\varepsilon_e$. The high case for $\varepsilon_e$ is $\varepsilon_e = 3$ and the low case for $\varepsilon_e$ is $\varepsilon_e = 1.1$. For each perturbation of the eight parameters I did not pin down using the oil shocks, $\{\varepsilon_y, \phi, \varepsilon_e, \alpha_f, \alpha_n, \rho_f, \rho_g, S\}$, I recalibrate the remaining six parameters $\{\alpha_g, \eta, \gamma, \varepsilon_f, \delta_y, \delta_{\widetilde{F}}\}$ and the productivity shock, $\nu$, to ensure that the model matches the moments described in Section IV.

Given the prominence of $\varepsilon_e$ in the literature, I conduct a sensitivity analysis over a wider range of values than for the other parameters in the model. Specifically, $\varepsilon_e$ ranges from 1.1 to 3. There is reasonable consensus in the literature that fossil and green energy are substitutes, suggesting that this elasticity should exceed unity. Empirical estimates from Lanzi and Sue Wing (2010) and Papageorgiou, Saam, and Schulte (2013) range from 1.6-3. The model is not consistent with the targeted moments for values of this elasticity greater than 3.

The first two blocks of Table F1 report the robustness analysis for the eight parameters that I did not pin down using the 1970s oil shocks, $\{\varepsilon_y, \phi, \varepsilon_e, \alpha_f, \alpha_n, \rho_f, \rho_g, S\}$. The third block of Table F1 reports robustness analysis for the six parameters that I calibrated from the 1970s oil shocks. Since the original values of these parameters were determined by the method-of-moments procedure, the model will not match the targeted moments for the perturbations of these parameter values.

The results are particularly sensitive to changes in the diminishing returns to innovation, $\eta$. Weaker diminishing returns (bigger $\eta$) increase the amount that agents raise green innovation in response to the carbon tax, thus increasing the effect of endogenous innovation on the size of the carbon tax.

The results are surprisingly insensitive to changes in the substitution elasticity between green energy and the composite comprised of fossil energy and foreign oil, $\varepsilon_e$. All else constant, lower values of $\varepsilon_e$ reduce the shift in demand from fossil to green energy in response to the tax. A smaller demand shift leads to a smaller change in innovation, decreasing the effects of endogenous innovation on the size of the carbon tax. However the magnitude of this decrease is reasonably small; even if $\widetilde{F}$ and $G$ are almost Cobb-Douglas ($\varepsilon_e = 1.1$), endogenous innovation still reduces the size of the carbon tax by 17.4 percent. The reason for this small effect is that matching the targeted moments with lower values of $\varepsilon_e$ requires weaker diminishing returns to innovation. As the strength of the diminishing returns to innovation falls ($\eta$ approaches unity), the effects of endogenous innovation on the size of the carbon tax increase, partially offsetting the decrease from the smaller substitution elasticity. Thus, the effects of changes in $\varepsilon_e$ are smaller than one might expect when the model is required to match the historical record.

Changes in the machine share in the fossil and green energy sectors have non-monotonic implications for the size of the required carbon tax. This is because the machine share has two potentially offsetting effects on the returns to innovation in a sector. First, from equation (A6), a decrease in $\alpha_f$ reduces fossil energy machine demand, lowering $X_{fit}$ and the marginal returns to innovation (equation (A10)). Working in the other direction, a decrease in $\alpha_f$ reduces the price elasticity of demand for machines, raising the optimal markup (equation (A9)) and the returns to innovation (equation (A10)). Which of these effects dominates depends on the region of the parameter space and on the model's general equilibrium channels (which effect the other endogenously determined quantities in equation (A10)). Thus, the sensitivity results are not always monotonic with respect to the machine share.

The low case for the machine share in non-energy implies only a 4.9 percent change in the carbon tax from endogenous innovation, the smallest response in the table. While at first this result might seem surprising, matching the moments with the low value of $\alpha_n$ requires very strong diminishing returns ($\eta = 0.32$), which substantially reduce the importance of endogenous innovation.

Increases in $\phi$, the magnitude of the across-sector spillovers, dampen the response of innovation to the carbon tax, reducing the role of endogenous innovation. However, this effect is relatively small. Online Appendix F.F2 extends this robustness analysis to include values of $\phi$ ranging from 0.3 to 0.9. Even with very large spillovers, ($\phi = 0.9$) endogenous innovation still reduces the carbon tax by over 15 percent.

Increases in the distribution parameter, $\delta_{\widetilde{F}}$ reduce the role of endogenous innovation. Higher values of $\delta_{\widetilde{F}}$ increase the final good producer's demand for fossil energy relative to oil imports, (see equation (A2) in online Appendix A). Since the carbon content of fossil energy is smaller than that of oil imports, this shift towards fossil energy results in a smaller increase in the after-tax price of $\widetilde{F}$, the composite comprised fossil energy and oil imports from the carbon tax. The smaller after-tax price change of $\widetilde{F}$ reduces the increase in green innovation in response to the tax.

Higher values of $\gamma$ increase the effect of changes in innovation on the relative levels of technology, thereby increasing the role of endogenous innovation. Finally, changes in the number of scientists, sector diversity and output distribution parameter all have relatively small affects on the role of endogenous innovation.

### *F2. Cross-sector spillovers: $\phi$*

I reexamine the main results when $\phi$ ranges from 0.3 to 0.9, the values for which there exists a stable interior balanced growth path in which agents innovate in both fossil and green energy. For each value of $\phi$, I recalibrate the six parameters $\{\alpha_g, \eta, \gamma, \varepsilon_f, \delta_y, \delta_{\tilde{F}}\}$ and the productivity shock, $\nu$, to ensure that the model matches the moments described in Section IV. Additionally, I recalculate the carbon tax that is required to achieve the 30 percent reduction in emissions in 20 years.

The left panel of Figure F1 plots the ratio of green to fossil technology after 20 years as a function of $\phi$ and the right panel plots the percent reduction in the carbon tax from endogenous innovation. Stronger spillovers decrease the change in relative technologies and, thus, reduce the effects of endogenous innovation on the size of the tax. However, even for very large spillovers, $\phi = 0.9$, endogenous innovation still reduces the size of the carbon tax by over 15 percent.

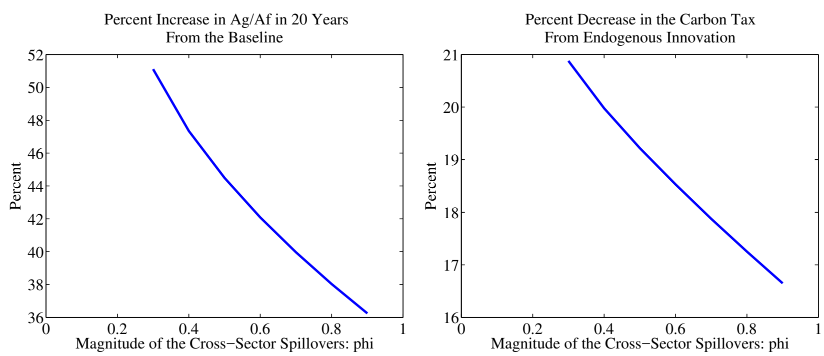


FIGURE F1. EFFECTS OF $\phi$ ON CHANGES IN RELATIVE TECHNOLOGY AND ON THE CARBON TAX

### *F3. Level of the carbon tax*

Tables F2 and F3 report the levels of the carbon tax requires to achieve the target for each parameter perturbation in Table F1 in online Appendix F.F1. In most of the perturbations, the level of the carbon tax is relatively similar to the level used in the main analysis. The largest differences occur for changes in $\varepsilon_e$, the elasticity of substitution between green energy and the composite comprised of fossil energy and oil imports. Some of this effect is due to the larger range values for $\varepsilon_e$; the low and high cases correspond to $\varepsilon_e = 1.1$ and 3, respectively as opposed to the 25 percent changes from the central value used for the other parameters. Even so, the result is intuitive; as the substitutability between these energy sources increases, the size of the carbon tax necessary to achieve the emissions target falls.

Table F2—Carbon Tax With Endogenous Innovation (in 2013 $ per ton $CO_2$)

| Parameter | High | Central | Low |
|---|---|---|---|
| *Imposed parameters* | | | |
| Output elasticity of substitution: $\varepsilon_y$ | 24.5 | 24.5 | - |
| Cross-sector spillovers: $\phi$ | 24.5 | 24.5 | 24.5 |
| *Direct from data series* | | | |
| Machine share in fossil energy: $\alpha_f$ | 25.6 | 24.5 | 28.9 |
| Machine share in non-energy: $\alpha_n$ | 22.4 | 24.5 | 28.3 |
| Energy elasticity of substitution: $\varepsilon_e$ | 8.4 | 24.5 | 40.6 |
| Sector diversity: $\rho_f$ | 24.9 | 24.5 | 23.9 |
| Sector diversity: $\rho_g$ | 24.3 | 24.5 | 24.6 |
| Population of scientists: $S$ | 24.5 | 24.5 | 24.5 |
| *Method of moments* | | | |
| Machine share in green energy: $\alpha_g$ | 24.1 | 24.5 | 20.4 |
| Diminishing returns: $\eta$ | 19.6 | 24.5 | 26.7 |
| Fossil elasticity of substitution: $\varepsilon_f$ | 25.4 | 24.5 | 23.5 |
| Distribution parameter: $\delta_y$ | 24.5 | 24.5 | 24.4 |
| Distribution parameter: $\delta_{\widetilde{F}}$ | 28.2 | 24.5 | 15.6 |

*Note:* The central column reports the result when the parameter equals its calibrated value in the main specification (given in Table 1). The high (low) column reports the result when the parameter is 25 percent bigger (smaller) then its value in Table 1. The one exception to this rule is for $\varepsilon_e$. The high case for $\varepsilon_e$ is $\varepsilon_e = 3$ and the low case for $\varepsilon_e$ is $\varepsilon_e = 1.1$. For each perturbation of the eight parameters I did not pin down using the oil shocks, $\{\varepsilon_y, \phi, \varepsilon_e, \alpha_f, \alpha_n, \rho_f, \rho_g, S\}$, I recalibrate the remaining six parameters $\{\alpha_g, \eta, \gamma, \varepsilon_f, \delta_y, \delta_{\widetilde{F}}\}$ and the productivity shock, $\nu$, to ensure that the model matches the moments described in Section IV.

### *F4. Price elasticity of green ideas*

Table F4 analyzes an alternative summary statistic for the different parameter perturbations, the price elasticity of green ideas defined in equations (13) and (14) in the text. As discussed in Section IV.E, the price elasticity of green ideas implied by the quantitative model is larger than the empirical estimates in Popp (2002) but smaller than the empirical estimates in Aghion et al. (2016).

This elasticity is endogenous to the model. However, the parameter perturba-

Table F3—Carbon Tax With Exogenous Innovation (in 2013 \$ per ton $CO_2$)

| Parameter | High | Central | Low |
|---|---|---|---|
| *Imposed parameters* | | | |
| Output elasticity of substitution: $\varepsilon_y$ | 30.2 | 30.3 | - |
| Cross-sector spillovers: $\phi$ | 30.0 | 24.5 | 30.7 |
| *Direct from data series* | | | |
| Machine share in fossil energy: $\alpha_f$ | 29.5 | 30.3 | 31.7 |
| Machine share in non-energy: $\alpha_n$ | 30.4 | 30.3 | 29.8 |
| Energy elasticity of substitution: $\varepsilon_e$ | 10.5 | 30.3 | 49.2 |
| Sector diversity: $\rho_f$ | 30.9 | 30.3 | 29.5 |
| Sector diversity: $\rho_g$ | 30.1 | 30.3 | 30.5 |
| Population of scientists: $S$ | 30.3 | 30.3 | 30.3 |
| *Method of moments* | | | |
| Machine share in green energy: $\alpha_g$ | 24.1 | 30.3 | 27.8 |
| Diminishing returns: $\eta$ | 30.6 | 30.3 | 29.9 |
| Fossil elasticity of substitution: $\varepsilon_f$ | 31.2 | 30.3 | 29.4 |
| Distribution parameter: $\delta_y$ | 30.3 | 30.3 | 30.2 |
| Distribution parameter: $\delta_{\widetilde{F}}$ | 33.9 | 30.3 | 23.2 |

*Note:* The central column reports the result when the parameter equals its calibrated value in the main specification (given in Table 1). The high (low) column reports the result when the parameter is 25 percent bigger (smaller) then its value in Table 1. The one exception to this rule is for $\varepsilon_e$. The high case for $\varepsilon_e$ is $\varepsilon_e = 3$ and the low case for $\varepsilon_e$ is $\varepsilon_e = 1.1$. For each perturbation of the eight parameters I did not pin down using the oil shocks, $\{\varepsilon_y, \phi, \varepsilon_e, \alpha_f, \alpha_n, \rho_f, \rho_g, S\}$, I recalibrate the remaining six parameters $\{\alpha_g, \eta, \gamma, \varepsilon_f, \delta_y, \delta_{\widetilde{F}}\}$ and the productivity shock, $\nu$, to ensure that the model matches the moments described in Section IV.

tions of the non-energy machine share, $\alpha_n$ can bring the elasticity close to the empirical values estimated in Popp (2002) and Aghion et al. (2016). In the high case for $\alpha_n$, the elasticity is 3.2, smaller than but close to Aghion et al. (2016)'s value of 3.7. In the low case, the elasticity is 0.2, approximately equal to Popp (2002)'s value of 0.21. From Table F1, the percent change in the carbon tax from endogenous innovation is 26.2 in the high case for $\alpha_n$ and is 4.9 in the low case. Thus, an elasticity closer to Aghion et al. (2016)'s value would increase the importance of endogenous innovation while an elasticity closer to Popp (2002)'s value would decrease its importance.[7]

[7]The large changes in the price elasticity of green ideas in the high and low cases for $\alpha_f$ and $\alpha_n$ arise because recalibrating the model with alternative values for these parameters results in substantially different values for the diminishing returns to innovation, $\eta$. Similarly, the low elasticity when $\varepsilon_e = 3$ occurs because matching the targeted moments with the high value of $\varepsilon_e$ requires much stronger diminishing returns to innovation than in the main specification.

Table F4—Price Elasticity of Green Ideas

| Parameter | High | Central | Low |
|---|---|---|---|
| *Imposed parameters* | | | |
| Output elasticity of substitution: $\varepsilon_y$ | 1.7 | 1.7 | - |
| Cross-sector spillovers: $\phi$ | 1.6 | 1.7 | 1.8 |
| *Direct from data series* | | | |
| Machine share in fossil energy: $\alpha_f$ | 5.2 | 1.7 | 0.3 |
| Machine share in non-energy: $\alpha_n$ | 3.2 | 1.7 | 0.2 |
| Energy elasticity of substitution: $\varepsilon_e$ | 0.5 | 1.7 | 1.6 |
| Sector diversity: $\rho_f$ | 1.8 | 1.7 | 1.6 |
| Sector diversity: $\rho_g$ | 1.7 | 1.7 | 1.7 |
| Population of scientists: $S$ | 1.7 | 1.7 | 1.7 |
| *Method of moments* | | | |
| Machine share in green energy: $\alpha_g$ | 1.7 | 1.7 | 1.4 |
| Diminishing returns: $\eta$ | 5.6 | 1.7 | 0.7 |
| Scientist efficiency: $\gamma$ | 1.8 | 1.7 | 1.7 |
| Fossil elasticity of substitution: $\varepsilon_f$ | 1.7 | 1.7 | 1.7 |
| Distribution parameter: $\delta_y$ | 1.7 | 1.7 | 1.7 |
| Distribution parameter: $\delta_{\widetilde{F}}$ | 1.9 | 1.7 | 1.5 |

*Note:* The central column reports the result when the parameter equals its calibrated value in the main specification (given in Table 1). The high (low) column reports the result when the parameter is 25 percent bigger (smaller) then its value in Table 1. The one exception to this rule is for $\varepsilon_e$. The high case for $\varepsilon_e$ is $\varepsilon_e = 3$ and the low case for $\varepsilon_e$ is $\varepsilon_e = 1.1$. For each perturbation of the eight parameters I did not pin down using the oil shocks, $\{\varepsilon_y, \phi, \varepsilon_e, \alpha_f, \alpha_n, \rho_f, \rho_g, S\}$, I recalibrate the remaining six parameters $\{\alpha_g, \eta, \gamma, \varepsilon_f, \delta_y, \delta_{\widetilde{F}}\}$ and the productivity shock, $\nu$, to ensure that the model matches the moments described in Section IV.

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